Pergamon

Int. J. Heat Mass Transfer. Vol. 37, No. 16, pp. 2588–2591, 1994

0017-9310/94 $7.00+0.00

Film boiling heat transfer for saturated drops impinging on a heating surface

SHIGEAKI INADA

Department of Mechanical System Engineering, Gunma University, Kiryu 376, Japan

and

WEN-JEI YANG†

Department of Mechanical Engineering and Applied Mechanics, University of Michigan, Ann Arbor, MI 48109-2125, U.S.A.

(*Received* 16 *March* 1993 *and in final form* 28 *February* 1994)

INTRODUCTION

FILM boiling of liquid drops in contact with a heating surface can be found in combustion inside a fuel injection engine, spray cooling of a heating surface, mist flow inside a heating tube and vapor explosion at liquid–liquid contact. In general, heat transfer performance in the liquid-drop system is superior to that in pool boiling, and is thus characterized by broad applications. However, its mechanism is not well understood, since the dynamic behavior and transient heat transfer of drops are clearly interrelated. For example, to determine the heat transfer rate between a drop and a heating surface, it is necessary to determine the timewise variations of both the drop spreading area and vapor layer thickness, the vapor flow velocity and the effects of the impinging Weber number, drop diameter and wall superheat on the heat transfer rate.

The experimental studies on film boiling of drops include Wachters *et al.* [1], Pedersen [2], Ueda *et al.* [3] and Shoji *et al.* [4]. They determined drop heat transfer effectiveness, not from the measurement of heat transfer coefficients, but from the timewise variations in both the temperature and heat capacity of the heating surface following drop impingement. By approximating drop shape as a circular cylinder and a hemisphere, Kendall and Rohsenow [5] conducted heat transfer analysis taking into account both the motion due to drop deformation and the motion of the drop's gravitational center. They determined the radius of the spreading drop, vapor layer thickness, timewise variations in heat fluxes and the effects of the drop diameter, wall superheat and impinging Weber number on heat transfer effectivenes. The assumed drop geometry produced little difference in analytical results. No attempt was made to derive a dimensionless correlation equation indicating the effects of each governing parameter on heat transfer performance (i.e. Nusselt number). Hence, results for film boiling heat transfer of an impinging drop system (Kendall and Rohsenow [5]) are not applicable to broad ranges of the governing parameters.

The present study derives a dimensionless correlation equation for film boiling heat transfer of an impinging drop system, as a function of the Weber number related to the impinging velocity of a drop, Bond number, Prandtl number and a new parameter, that is, the Weber number related to the vapor stream, through an analytical investigation of five liquids with distinct physical properties. Results are compared with the empirical correlation equation for forced convection film boiling over a circular cylinder (Bromley *et al.* [6]) to determine the range of its applicability to the present drop boiling system.

† Author to whom correspondence should be addressed.

THEORETICAL ANALYSIS

Consider a vapor layer enclosed between a drop base and a heating surface, as shown in Fig. 1. The cylindrical coordinate system (r, z) is employed with the origin fixed at the center of the drop base. The coordinates r and z measure the radial and axial distance, respectively. u_r and u_z, respectively, denote the velocity components in the r and z directions. T represents the stream temperature. The conservation equations then read:

continuity equation:

$$\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_z}{\partial z}=0 \tag{1}$$

momentum equation in the r direction:

$$\rho_g\left(u_r\frac{\partial u_r}{\partial r}+u_z\frac{\partial u_r}{\partial z}\right)=-\frac{\partial P}{\partial r}+\mu_g\frac{\partial^2 u_r}{\partial z^2} \tag{2}$$

energy equation:

$$u_r\frac{\partial T}{\partial r}+u_z\frac{\partial T}{\partial z}=a_g\frac{\partial^2 T}{\partial z^2}. \tag{3}$$

The appropriate boundary conditions are:

at $z = 0$:

$$u_r = 0,\quad u_z = 0,\quad T = T_{w0},\quad \partial^2 T/\partial z^2 = 0 \tag{4}$$

at $z = z_b$:

$$u_r = U_r,\ u_z = -U_0,\ T = T_s,\ \partial T/\partial z = -\beta_2(T_{w0}-T_s)/z_b. \tag{5}$$

Here, U_r denotes the radial flow velocity within the drop (expressed by equation (16)) and U_0 is the vapor ejection velocity at the drop base whose vertical translation velocity is taken into account. z_b represents the vapor layer thickness ($z_b = z_c - W/2$) and β_2 is a nonlinear temperature distribution coefficient inside the vapor layer. After all the factors, β_1 and β_2 are introduced to account for the non-

NOMENCLATURE

a_g	thermal diffusitivity [$m^2 s^{-1}$]
Bo	Bond number, $g(\rho_f - \rho_g)D^2/\sigma$
D	drop diameter [m]
D_s	horizontal tube diameter [m]
F	force defined by equation (11) [N]
g	gravitational acceleration [$m s^{-2}$]
I_1, I_2	dimensionless parameters defined by equation (12)
L_g	latent heat of vaporization [$kJ kg^{-1}$]
Nu	Nusselt number
Pr	Prandtl number of liquid drop
Q_w	heat flow rate to drop [W]
q	heat flux [$W m^{-2}$]; q_w at heating surface; q_{ev} at drop base
R	radius of drop base [m]
r	radial coordinate [m]
S_c	spreading area of drop base [m^2]
T	temperature [°C]
T_h	Weber number for vapor stream
T_s	saturation temperature [°C]
T_{w0}	heating surface temperature [°C]
U_a	steam ejection velocity at drop base [$m s^{-1}$]
U_b	velocity of drop base [$m s^{-1}$]
U_0	net steam velocity at drop base, $U_a - U_b$ [$m s^{-1}$]
u	steam velocity [$m s^{-1}$]; u_r and u_z in the r and z directions, respectively
u_s	flow velocity crossed the horizontal tube [$m s^{-1}$]
V_0	fall velocity of liquid drop [$m s^{-1}$]
W	drop height [m]; W_0, initial height
We	Weber number
Z_0	initial height of drops gravitational center [m]
z	axial coordinate [m]; z_b of drop base; z_c of drop gravitational center.

Greek symbols

α	heat transfer coefficient for drop impinging film boiling [$W m^{-2} K^{-1}$]
α_{co}	heat transfer coefficient for forced convection film boiling [$W m^{-2} K^{-1}$]
β	nonlinear temperature distribution coefficient; β_1 at heated surface inside vapor layer; β_2 at drop base inside vapor layer
ΔT_{sat}	wall superheat, $T_{w0} - T_s$ [K]
λ_g	thermal conductivity [$W m^{-1} K^{-1}$]
μ_g	viscosity [Pa s]
ν_g	kinematic viscosity [$m^2 s^{-1}$]
ρ_g	density [$kg m^{-3}$]
σ	surface tension [$N m^{-1}$]
τ	time [s]
τ_c	effective heat transfer time [s]
τ_f	freely oscillating period of a liquid drop [s].

Subscripts

f	liquid phase
g	steam
0	initial value.

Superscript

$\bar{}$	time averaged value.

linearity in the temperature distribution, so that the heat flux removed from the wall is:

$$q_w = \beta_1 \lambda_g (T_{w0} - T_s)/z_b \tag{6}$$

and the heat flux transferred to the drop base is:

$$q_{ev} = \beta_2 \lambda_g (T_{w0} - T_s)/z_b. \tag{7}$$

Here, λ_g denotes the thermal conductivity of the vapor; T_{w0} is the heating surface temperature, and T_s is the saturation temperature.

If the boundary condition, $\partial T/\partial z = -\beta_1(T_{w0} - T_s)/z_b$ at $z = 0$ is applied, β_1 and β_2 can be related by

$$2\beta_1 + \beta_2 = 3 \tag{8}$$

where β_1 and β_2 are still unknown. These factors are determined by integrating equation (3) with respect to z.

For simplicity in mathematical manipulation, it is assumed that at the moment of impingement on the heating surface, a spherical drop of diameter D takes the form of a cylinder with radius R and height W, as illustrated in Fig. 1. The drop then changes in shape on the heating surface under the condition of constant volume, with its height W varying as a function of time τ. Let the height of its gravitational center be z_c. Then we can derive an equation for drop deformation based on the internal flow as

$$\frac{d^2W}{d\tau^2} = -\frac{6}{\pi D^3 \rho_f}\left[\frac{1}{2}F + 2\pi\sigma D\left(\frac{1}{2}\sqrt{\left(\frac{D}{6W}\right)} - \frac{1}{6}\left(\frac{D}{W}\right)^2\right) - \frac{\rho_f \pi D^3}{32D6}\left(\frac{D}{W}\right)^4\left(\frac{dW}{d\tau}\right)^2\right] \Big/ \left[\frac{1}{12} + \frac{1}{48}\left(\frac{D}{W}\right)^3\right] \tag{9}$$

$$\frac{d^2 z_c}{d\tau^2} = \frac{6F}{\pi D^3 \rho_f} - g. \tag{10}$$

Here, ρ_f, σ, g and F denote the liquid density, surface tension, gravitational acceleration, and force acting on the drop base, respectively.

The force F is induced by the pressure rise inside the superheated vapor layer formed between the drop base and the heating surface. The pressure rise results from the radial flow that overcomes the viscous force within the vapor layer and the accelerating flow in the radial direction out from between the drop and the wall. Hence, the force F can be derived from the continuity and momentum equations of the

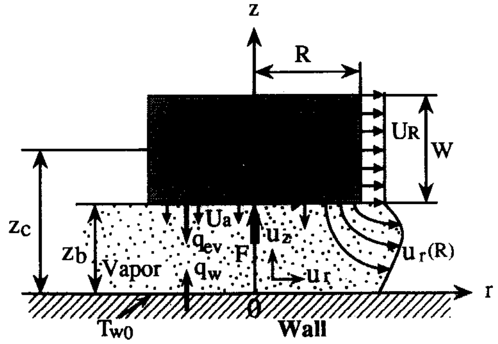

FIG. 1. Model of deforming drop and vapor flow.

vapor layer as

$$F = \frac{\pi R^4}{4}\left(\frac{\rho_g U_0^2}{z_b^2} I_1 - \frac{\mu_g U_0}{2z_b^3} I_2\right) \quad (11)$$

where

$$I_1 = 9/10 + 13/(20(U_w/U_0)) + 1/(10(U_w/U_0)^2) \quad (12a)$$

$$I_2 = -12 - 6(U_w/U_0). \quad (12b)$$

Here, ρ_g and μ_g are the density and viscosity of the superheated stream, respectively. U_0 represents the net vertical velocity of vapor at the drop base, and is the sum of the velocity of the drop base U_b, and the vapor ejection velocity at the drop base U_a. That is,

$$U_a = \beta_2 \lambda_g \Delta T_{sat}/(\rho_g L_g z_b) \quad (13)$$

$$U_b = dz_c/d\tau - 1/2(dW/d\tau) \quad (14)$$

$$U_0 = U_a + (-U_b) \quad (15)$$

$$U_r = (-1/2)(U_w/z_b)r \quad (16)$$

$$U_w = (dW/d\tau)(z_b/W) \quad (17)$$

where L_g represents the latent heat of evaporation and $\Delta T_{sat} = T_{w0} - T_s$.

The conservation equations (1)–(3) and the drop deformation equations (9) and (10) are coupled. The physical time is divided into small intervals. At each time interval, the coupled equations are integrated by means of the Runge–Kutta method.

The initial conditions are

$$\beta_2 = 1, \quad W = W_0 = 0.87D, \quad dW/d\tau = 0$$
$$z_c = Z_0, \quad dz_c/d\tau = -V_0. \quad (18)$$

Here, Z_0 is the initial height of the drop's gravitational center and may take an arbitrary value for which $z > W_0/2$. V_0 is the falling velocity of the drop.

The computational procedure is to first substitute the instantaneous values of W and z_c obtained from equations (9) and (10) into the momentum equation (2) to determine F, followed by evaluating β_2 from the energy equation (3). In the next time instant, the Runge–Kutta method is employed to integrate equations (9) and (10).

RESULTS AND DISCUSSION

Results are obtained for various liquid drops of water, methanol, ethanol, butanol and propanol. Drop diameters vary in the range 0.22, 1.0, 2.3 and 4.0 mm, wall superheats in the range 200, 400 and 600 K, and Weber numbers for impinging drop in the range 12.3, 25, 50, 100 and 300. The physical properties at the saturated film temperature $(T_{w0} + T_s)/2$ are employed in the numerical computations. The dimensionless parameters are defined as: Weber number for liquid drop, $We = \rho_l V_0^2 D/\sigma$; Bond number, $Bo = g(\rho_l - \rho_g)D^2/\sigma$; Weber number for vapor stream, that is, inertia force/surface tension force,

$$T_h = (\lambda_g \Delta T_{sat}/L_g)/\sqrt{(\rho_g \sigma D)} = \sqrt{(\rho_g U_a^2 z_b^2)}/\sqrt{(\sigma D)}$$

where U_a is expressed by equation (13) with $\beta_2 = 1$.

Figure 2 is a typical result illustrating the time history of q_w for butanol drops of $D = 4.0$ mm and $\Delta T_{sat} = 200$ K. The abscissa is the physical time non-dimensionalized by the freely oscillating period of a liquid drop τ_r. It is obvious in Fig. 2 that q_w reduces with an increase in Weber number. The heat flux q_w takes two to three maxima within the period. The time-averaged heat transfer performance (Nusselt number) is defined based on the time-averaged value of q_w as

$$Nu = \bar{q}_w/(\lambda_g \Delta T_{sat}/D) \quad (19)$$

where

$$\bar{q}_w = \bar{Q}_w/\bar{S}_c \quad (20)$$

$$\bar{Q}_w = \frac{1}{\tau_c}\int_0^{\tau_c} q_w \pi R^2 \, d\tau \quad (21)$$

$$\bar{S}_c = \frac{1}{\tau_c}\int_0^{\tau_c} \pi R^2 \, d\tau. \quad (22)$$

τ_c signifies the effective heat transfer time. It is almost equal to the restoring period for an impinging drop, the time it takes to reach the original value W_0. $\bar{q}_w$ reduces resulting from an enhancement in πR^2 (spreading area of the drop) in equation (22) as the Weber number is increased.

Figure 3 plots all data obtained in the present analysis. A correlation equation expressed by equation (23) is derived with a scattering range between +50% and −36%:

$$Nu = 2.80 T_h^{-0.56}(We/2 + 13)^{-0.56} Bo^{-0.056} Pr^{0.31}. \quad (23)$$

Here, Pr denotes the Prandtl number of the liquid. The effects of ΔT_{sat}, D and We on the heat transfer coefficient α are derived from equation (23) as

$$\alpha \propto \Delta T_{sat}^{-0.56} D^{-0.832}(We/2 + 13)^{-0.56}. \quad (24)$$

This shows that α is enhanced with a decrease in ΔT_{sat}, D and We.

To the authors' knowledge, there are no experimental data to ascertain the validity of the present analysis (represented by equation (23)). However, it is of interest to compare film boiling of saturated drops impinging on a heating surface with forced convection film boiling from a horizontal tube (Bromley *et al.* [6]) whose correlation equation is

$$\alpha_{co}\sqrt{[D_s \Delta T_{sat}/(u_s \lambda_g \rho_g L_g)]} = 2.7 \quad (25)$$

where α_{co} is the heat transfer for forced convection film boiling, D_s is the horizontal tube diameter and u_s is the flow velocity across the tube. Treating the tube diameter D_s as the drop diameter, and the flow velocity u_s as the velocity of a drop impinging on the heating surface, equation (25) can be rewritten in dimensionless form as

$$Nu = 2.7 T_h^{-0.5} We^{0.25}(\rho_g/\rho_l)^{0.25}. \quad (26)$$

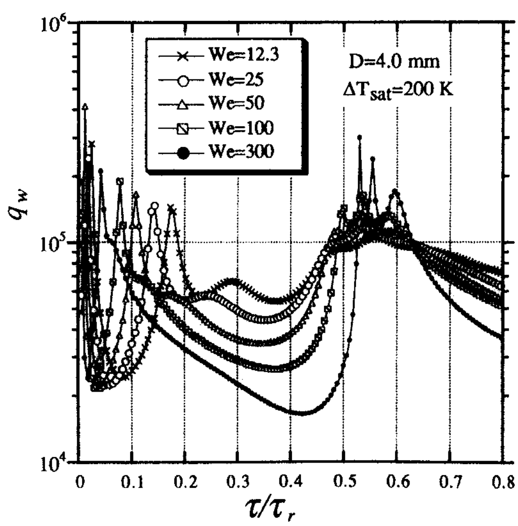

FIG. 2. Time history of heat flux for butanol drops.

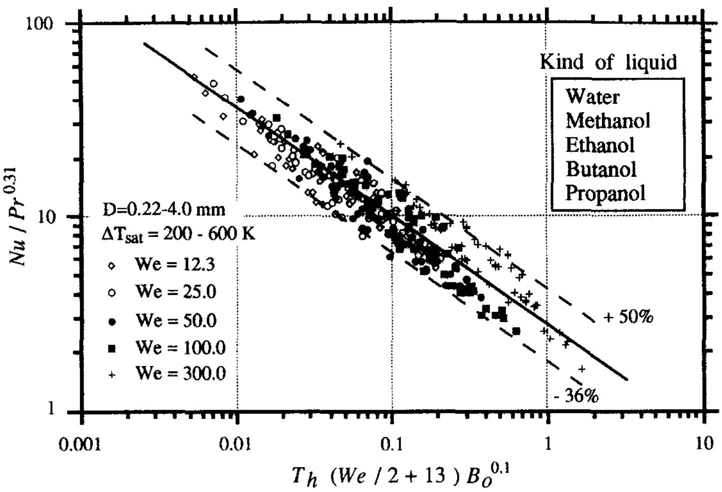

FIG. 3. A correlation of film boiling heat transfer for saturated drops impinging on a heating surface.

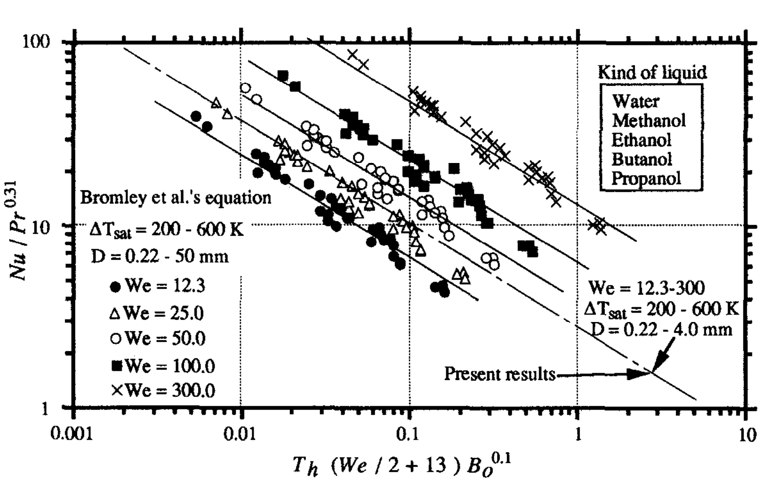

FIG. 4. Comparison of heat transfer between film boiling of saturated drops impinging on a heating surface and a forced convection film boiling over a horizontal tube.

Equation (26), together with test data of Bromley *et al.* [6], are plotted in Fig. 4 for $D = 0.22$–50 mm, $\Delta T_{sat} = 200$–600 K, and $We = 12.3$–300. Equation (23) is superimposed as the chain line for comparison. It is seen that equations (23) and (26) have the same slope, and that equation (26) is applicable to film boiling of saturated drops impinging on a heating surface for Weber number in the range of 12.3–50.

CONCLUSIONS

A theoretical analysis has been conducted to determine film boiling heat transfer of saturated drops impinging on a heated surface, taking into account the dynamics of drop deformation. A new dimensionless parameter has been derived which relates conductive heat transfer through a vapor layer, and evaporation rate of the liquid induced by surface tension. It is concluded from the study that the heat transfer performance of the drop film boiling system can be predicted by equation (23), and that the empirical correlation equation for forced convection film boiling over a circular cylinder, equation (26), can be applied to the drop film boiling system with the impinging Weber number between 12.3 and 50.

REFERENCES

1. L. H. J. Wachters, L. Smulders, J. R. Vermeulen and H. C. Kleiweg, The heat transfer from a hot wall to impinging mist droplets in the spheroidal state, *Chem. Engng Sci.* **21**, 1231–1238 (1966).
2. C. O. Pedersen, An experimental study of the dynamic behavior and heat transfer charactristics of water droplets impinging upon a heated surface, *Int. J. Heat Mass Transfer* **13**, 369–381 (1970).
3. T. Ueda, T. Enomoto and M. Kanetsuki, Heat transfer characteristics and dynamic behavior of saturated droplets impinging on a heated vertical surface, *Bull. JSME* **22**, 724–732 (1979).
4. M. Shoji, T. Wakunaga and K. Kodama, Heat transfer between subcooled impinging droplet and a heated surface (heat transfer characteristics in non-wetting region), *Trans. JSME* **50**, 716–723 (1984).
5. G. E. Kendall and W. M. Rohsenow, Heat transfer to impacting drops and post critical heat flux dispersed flow, MIT, Tech. Rep. No. 85694-100 (March 1978).
6. L. A. Bromley, N. R. Leroy and J. A. Robbers, Heat transfer in forced convection film boiling, *Industrial Engng Chem.* **45**, 2639–2646 (1953).